

### Learning Abstract:

This assignment is made up nine tasks which involve a mixture of using premade code in addition to deducing my own predicates to solve the classic Towers of Hanoi problem but in a Prolog context. In addition to this, this assignment is intended to further develop a conceptual understanding of state space problem solving. The demos included are for the 3- and 4-disc part of the problem. An interesting worthwhile side artifact is that when I attempted to run the problem as a 5-disc problem the program was not able to finitely define a solution. This is likely due in part to the fact that program is performing a blind search in which adding an additional disk yield in exponentially longer compute times with each addition of a disc. Different search methodologies could solve this problem .

### Task 1:

Contemplate the nature of the problem, see specification on web page for details.

### Task 2:

Copy and paste source code and check to ensure validity and that it initially compiles, see specification on web page for details, full code posted later this document.

### Task 3: One Move Predicate and a Unit Test

```
m12([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].  
  
test_m12 :-  
    write('Testing: move_m12\n'),  
    TowersBefore = [[t,s,m,l,h],[],[ ]],  
    trace(' ', 'TowersBefore', TowersBefore),  
    m12(TowersBefore, TowersAfter),  
    trace(' ', 'TowersAfter', TowersAfter).
```

```
$ swipl  
Welcome to SWI-Prolog (threaded, 64 bits, version 8.5.8-154-g70a18c809)  
SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software.  
Please run ?- license. for legal details.  
  
For online help and background, visit https://www.swi-prolog.org  
For built-in help, use ?- help(Topic). or ?- apropos(Word).  
  
1 ?- consult('toh.pro').  
true.  
  
2 ?- test_m12.  
Testing: move_m12  
TowersBefore = [[t,s,m,l,h],[],[ ]]  
TowersAfter = [[s,m,l,h],[t],[ ]]  
true.  
  
3 ?- █
```

#### Task 4: The Remaining Five Move Predicates and a Unit Tests

```
m12([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].  
  
m13([Tower1Before, Tower2, Tower3Before], [Tower1After, Tower2, Tower3After]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower3Before = L,  
    Tower3After = [H|L].  
  
m21([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower2Before = [H|T],  
    Tower2After = T,  
    Tower1Before = L,  
    Tower1After = [H|L].  
  
m23([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-  
    Tower2Before = [H|T],  
    Tower2After = T,  
    Tower3Before = L,  
    Tower3After = [H|L].  
  
m31([Tower1Before, Tower2, Tower3Before], [Tower1After, Tower2, Tower3After]) :-  
    Tower3Before = [H|T],  
    Tower3After = T,  
    Tower1Before = L,  
    Tower1After = [H|L].  
  
m32([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-  
    Tower3Before = [H|T],  
    Tower3After = T,  
    Tower2Before = L,
```

```
Tower2After = [H|L].
```

```
% --- Unit test programs

test__m12 :-
    write('Testing: move_m12\n'),
    TowersBefore = [[t,s,m,l,h],[],[ ]],
    trace(' ','TowersBefore',TowersBefore),
    m12(TowersBefore,TowersAfter),
    trace(' ','TowersAfter',TowersAfter).

test__m13 :-
    write('Testing: move_m13\n'),
    TowersBefore = [[t,s,m,l,h],[],[ ]],
    trace(' ','TowersBefore',TowersBefore),
    m13(TowersBefore,TowersAfter),
    trace(' ','TowersAfter',TowersAfter).

test__m21 :-
    write('Testing: move_m21\n'),
    TowersBefore = [[],[t,s,m,l,h],[ ]],
    trace(' ','TowersBefore',TowersBefore),
    m21(TowersBefore,TowersAfter),
    trace(' ','TowersAfter',TowersAfter).

test__m23 :-
    write('Testing: move_m23\n'),
    TowersBefore = [[],[t,s,m,l,h],[ ]],
    trace(' ','TowersBefore',TowersBefore),
    m23(TowersBefore,TowersAfter),
    trace(' ','TowersAfter',TowersAfter).

test__m31 :-
    write('Testing: move_m31\n'),
    TowersBefore = [[],[ ],[t,s,m,l,h]],
    trace(' ','TowersBefore',TowersBefore),
    m31(TowersBefore,TowersAfter),
    trace(' ','TowersAfter',TowersAfter).

test__m32 :-
    write('Testing: move_m32\n'),
    TowersBefore = [[],[ ],[t,s,m,l,h]],
    trace(' ','TowersBefore',TowersBefore),
    m32(TowersBefore,TowersAfter),
    trace(' ','TowersAfter',TowersAfter).
```

```
1 ?- consult('toh.pro').
true.

2 ?- test__m12.
Testing: move_m12
TowersBefore = [[t,s,m,l,h],[],[ ]]
TowersAfter = [[s,m,l,h],[t],[ ]]
true.

3 ?- test__m13.
Testing: move_m13
TowersBefore = [[t,s,m,l,h],[],[ ]]
TowersAfter = [[s,m,l,h],[],[t]]
true.

4 ?- test__m23.
Testing: move_m23
TowersBefore = [[],[t,s,m,l,h],[ ]]
TowersAfter = [[],[s,m,l,h],[t]]
true.

5 ?- test__m31.
Testing: move_m31
TowersBefore = [[],[],[t,s,m,l,h]]
TowersAfter = [[t],[],[s,m,l,h]]
true.

6 ?- test__m32.
Testing: move_m32
TowersBefore = [[],[],[t,s,m,l,h]]
TowersAfter = [[],[t],[s,m,l,h]]
true.

7 ?- test_m21.
Correct to: "test__m21"?
Please answer 'y' or 'n'? yes
Testing: move_m21
TowersBefore = [[],[t,s,m,l,h],[ ]]
TowersAfter = [[t],[s,m,l,h],[ ]]
true.

8 ?- █
```

### Task 5: Valid State Predicate and Unit Test

```
% -----  
% --- valid_state(S) :: S is a valid state  
  
valid_state([A|[B|[C]]]) :- towerState(A), towerState(B), towerState(C).  
  
towerState([]).  
towerState([s]).  
towerState([s,m]).  
towerState([s,m,l]).  
towerState([s,l]).  
towerState([s,l,h]).  
towerState([s,h]).  
towerState([s,m,h]).  
towerState([m]).  
towerState([m,l]).  
towerState([m,l,h]).  
towerState([m,h]).  
towerState([m]).  
towerState([m,l]).  
towerState([m,l,h]).  
towerState([m,h]).  
towerState([l]).  
towerState([l,h]).  
towerState([h]).  
towerState([s,m,l,h]).  
towerState([t]).  
towerState([t,s]).  
towerState([t,s,m]).  
towerState([t,s,m,l]).  
towerState([t,s,l]).  
towerState([t,s,l,h]).  
towerState([t,s,h]).  
towerState([t,s,m,h]).  
towerState([t,m]).  
towerState([t,m,l]).  
towerState([t,m,l,h]).  
towerState([t,m,h]).  
towerState([t,m]).  
towerState([t,m,l]).  
towerState([t,m,l,h]).  
towerState([t,m,h]).  
towerState([t,l]).
```

```
towerState([t,l,h]).
towerState([t,h]).
towerState([t,s,m,l,h]).

%% Unit Test Code

test_valid_state :-
    write('Testing: valid_state\n'),
    test_vs([[1,t,s,m,h],[],[[]]),
    test_vs([[t,s,m,l,h],[],[[]]),
    test_vs([[],[h,t,s,m],[1]]),
    test_vs([[],[t,s,m,h],[1]]),
    test_vs([[],[h],[1,m,s,t]]),
    test_vs([[],[h],[t,s,m,l]]).

test_vs(S) :-
    valid_state(S),
    write(S), write(' is valid. '), nl.

test_vs(S) :-
    write(S), write(' is invalid. '), nl.
```

```
4 ?- test_valid_state.
Testing: valid_state
[[1,t,s,m,h],[],[[]] is invalid.
[[t,s,m,l,h],[],[[]] is valid.
[[],[h,t,s,m],[1]] is invalid.
[[],[t,s,m,h],[1]] is valid.
[[],[h],[1,m,s,t]] is invalid.
[[],[h],[t,s,m,l]] is valid.
true
```

#### Task 6: Defining the write sequence predicate

```
%% Write Sequence Doe

write_sequence([]).
write_sequence([H|T]) :-
    elaborate(H,E),
    write(E),nl,
    write_sequence(T).
```

```
elaborate(m12,Output) :-  
    Output = 'Transfer a disk from tower 1 to tower 2.'  
  
elaborate(m13,Output) :-  
    Output = 'Transfer a disk from tower 1 to tower 3.'  
  
elaborate(m21,Output) :-  
    Output = 'Transfer a disk from tower 2 to tower 1.'  
  
elaborate(m23,Output) :-  
    Output = 'Transfer a disk from tower 2 to tower 3.'  
  
elaborate(m31,Output) :-  
    Output = 'Transfer a disk from tower 3 to tower 1.'  
  
elaborate(m32,Output) :-  
    Output = 'Transfer a disk from tower 3 to tower 2.'  
  
%% Unit Test Code  
  
test_write_sequence :-  
    write('First test of write_sequence ...'), nl,  
    write_sequence([m31,m12,m13,m21]),  
    write('Second test of write_sequence ...'), nl,  
    write_sequence([m13,m12,m32,m13,m21,m23,m13]).
```

```
5 ?- test_write_sequence.  
First test of write_sequence ...  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Second test of write_sequence ...  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 2.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 2 to tower 3.  
Transfer a disk from tower 1 to tower 3.  
true.
```

```
6 ?- █
```

### Task 7: Intermediate and Plain English Demo

```
3 ?- reconsult('toh.pro').
true.

4 ?- solve.
PathSoFar = [[s,m,1],[],[[]]]
Move = m12
NextState = [[m,1],[s],[[]]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]]]]
Move = m12
NextState = [[1],[m,s],[[]]]
Checking Valid State
Move = m13
NextState = [[1],[s],[m]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]]]
Move = m12
NextState = [[],[1,s],[m]]
Checking Valid State
Move = m13
NextState = [[],[s],[1,m]]
Checking Valid State
Move = m21
NextState = [[s,1],[],[m]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]],[[s,1],[],[m]]]
Move = m12
NextState = [[1],[s],[m]]
Move = m13
NextState = [[1],[],[s,m]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]]]
Move = m12
NextState = [[],[1],[s,m]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]],[[1],[],[s,m]],[[1],[],[s,m]]]
Move = m21
NextState = [[1],[],[s,m]]
Move = m23
NextState = [[],[],[1,s,m]]
Checking Valid State
Move = m31
NextState = [[s],[1],[m]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]],[[1],[],[s,m]],[[s],[1],[m]]]
Move = m12
NextState = [[],[s,1],[m]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]],[[1],[],[s,m]],[[s],[1],[m]],[[1],[s,1],[m]]]
Move = m21
NextState = [[s],[1],[m]]
Move = m23
NextState = [[],[1],[s,m]]
Move = m31
NextState = [[m],[s,1],[[]]]
Checking Valid State
PathSoFar = [[s,m,1],[],[[]],[[m,1],[s],[[]],[1],[s],[m]],[[s,1],[],[m]],[[1],[],[s,m]],[[1],[],[s,m]],[[s],[1],[m]],[[1],[s,1],[m]],[[m],[s,1],[[]]]]
Move = m12
NextState = [[],[m,s,1],[[]]]
Checking Valid State
Move = m13
NextState = [[],[s,1],[m]]
```

```

NextState = [[],[m,s,1],[ ]
Checking Valid State
Move = m13
NextState = [[],[s,1],[m]]
Move = m21
NextState = [[s,m],[1],[ ]
Checking Valid State
PathSoFar = [[[s,m,1],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]]]
Move = m12
NextState = [[m],[s,1],[ ]
Move = m13
NextState = [[m],[1],[s]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ]],[[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]]],[[ ]],[m,1],[s]]]
Move = m12
NextState = [[],[m,1],[s]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ]],[[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]]],[[ ]],[m,1],[s]]]
Move = m21
NextState = [[m],[1],[s]]
Move = m23
NextState = [[],[1],[m,s]]
Checking Valid State
Move = m31
NextState = [[s],[m,1],[ ]
Checking Valid State
PathSoFar = [[[s,m,1],[ ]],[[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]]],[[ ]],[m,1],[s]]],[[s],[m,1],[ ]]]
Move = m12
NextState = [[],[s,m,1],[ ]
Checking Valid State
PathSoFar = [[[s,m,1],[ ]],[[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]]],[[ ]],[m,1],[s]]],[[s],[m,1],[ ]],[[ ]],[s,m,1],[ ]]]
Move = m21
NextState = [[s],[m,1],[ ]
Move = m23
NextState = [[],[m,1],[s]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
PathSoFar = [[[s,m,1],[ ]],[[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]]],[[ ]],[m,1],[s]]],[[s],[m,1],[ ]]]
Move = m12
NextState = [[],[s,m,1],[ ]
Checking Valid State
PathSoFar = [[[s,m,1],[ ]],[[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ]],[m]],[[1],[ ]],[s,m]],[[ ]],[1],[s,m]],[[s],[1],[m]],[[ ]],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]]],[[ ]],[m,1],[s]]],[[s],[m,1],[ ]],[[ ]],[s,m,1],[ ]]]
Move = m21
NextState = [[s],[m,1],[ ]
Move = m23
NextState = [[],[m,1],[s]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]
Checking Valid State
Move = m23

```

```
Move = m21
NextState = [[s],[m,1],[ ]]
Move = m23
NextState = [[],[m,1],[s]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
Move = m32
NextState = [[],[s,m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[ ],[s,m,1],[ ]]]]
Move = m21
NextState = [[s],[m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[ ],[s,m,1],[ ]],[[s],[m,1],[ ]]]]
Move = m12
NextState = [[],[s,m,1],[ ]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
PathSoFar = [[[s,m,1],[ ],[]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[ ],[s,m,1],[ ]],[[s],[m,1],[ ]]]]
Move = m12
NextState = [[],[s,m,1],[ ]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
Move = m23
NextState = [[],[m,1],[s]]
PathSoFar = [[[s,m,1],[ ],[]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[ ],[s,m,1],[ ]],[[s],[m,1],[ ]]]]
Move = m21
NextState = [[m],[1],[s]]
Move = m23
NextState = [[],[1],[m,s]]
Checking Valid State
Move = m31
NextState = [[s],[m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]]]]
Move = m12
NextState = [[],[s,m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]],[[ ],[s,m,1],[ ]]]]
Move = m21
NextState = [[s],[m,1],[ ]]
Move = m23
NextState = [[],[m,1],[s]]
```

```
Move = m21
NextState = [[s],[m,1],[ ]]
Move = m23
NextState = [[],[m,1],[s]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
PathSoFar = [[[s,m,1],[ ],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]]]]
Move = m12
NextState = [[],[s,m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]],[[ ],[s,m,1],[ ]]]]
Move = m21
NextState = [[s],[m,1],[ ]]
Move = m23
NextState = [[],[m,1],[s]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
Move = m32
NextState = [[],[s,m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]],[[ ],[s,m,1],[ ]]]]
Move = m21
NextState = [[s],[m,1],[ ]]
Checking Valid State
PathSoFar = [[[s,m,1],[ ],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]],[[s],[m,1],[ ]]]]
Move = m12
NextState = [[],[s,m,1],[ ]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
Move = m23
NextState = [[],[m,1],[s]]
PathSoFar = [[[s,m,1],[ ],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]],[[s],[m,1],[ ]]]]
Move = m12
NextState = [[],[s,m,1],[ ]]
Move = m13
NextState = [[],[m,1],[s]]
Move = m21
NextState = [[m,s],[1],[ ]]
Checking Valid State
Move = m23
NextState = [[s],[1],[m]]
Move = m23
NextState = [[],[m,1],[s]]
PathSoFar = [[[s,m,1],[ ],[ ]],[[m,1],[s],[ ]],[[1],[s],[m]],[[s,1],[ ],[m]],[[1],[ ],[s,m]],[[ ],[1],[s,m]],[[s],[1],[m]],[[ ],[s,1],[m]],[[m],[s,1],[ ]],[[s,m],[1],[ ]],[[m],[1],[s]],[[ ],[m,1],[s]],[[s],[m,1],[ ]],[[s],[m,1],[ ]]]]
Move = m21
```



## Questions and Answers

1. *What was the length of your program's solutions to the three-disk problem?*

The length appears to be a degree of 14 steps.

2. *What is the length of the shortest solution to the three-disk problem?*

Doing it by hand I was able to complete it in 7 steps with three discs.

3. *How do you account for the discrepancy?*

It appears the program is checking each possible state before executing another transition bringing into question the computational efficiency of this process.

## Task 8:

```
Solution ...  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 3 to tower 1.  
Transfer a disk from tower 1 to tower 2.  
Transfer a disk from tower 1 to tower 3.  
Transfer a disk from tower 2 to tower 1.  
Transfer a disk from tower 1 to tower 3.
```

## Questions and Answers

1. *What was the length of your program's solutions to the four-disk problem?*

The length appears to be a degree of 40 steps.

2. *What is the length of the shortest solution to the four-disk problem?*

Doing it by hand I was able to complete it in 15 steps with four discs.

## Task 9: The Full Code Base

```
% -----  
% -----  
% --- File: towers_of_hanoi.pro  
% --- Line: Program to solve the Towers of Hanoi problem  
% -----  
  
:- consult('inspectors.pro').  
  
% -----  
% --- make_move(S,T,SSO) :: Make a move from state S to state T by SSO  
make_move(TowersBeforeMove,TowersAfterMove,m12) :-  
    m12(TowersBeforeMove,TowersAfterMove).  
  
make_move(TowersBeforeMove,TowersAfterMove,m13) :-  
    m13(TowersBeforeMove,TowersAfterMove).  
  
make_move(TowersBeforeMove,TowersAfterMove,m21) :-  
    m21(TowersBeforeMove,TowersAfterMove).  
  
make_move(TowersBeforeMove,TowersAfterMove,m23) :-  
    m23(TowersBeforeMove,TowersAfterMove).  
  
make_move(TowersBeforeMove,TowersAfterMove,m31) :-  
    m31(TowersBeforeMove,TowersAfterMove).  
  
make_move(TowersBeforeMove,TowersAfterMove,m32) :-  
    m32(TowersBeforeMove,TowersAfterMove).  
  
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-  
    Tower1Before = [H|T],  
    Tower1After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].  
  
m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-  
    Tower1Before = [H|T],
```

```
Tower1After = T,  
Tower3Before = L,  
Tower3After = [H|L].  
  
m21([Tower1Before, Tower2Before, Tower3], [Tower1After, Tower2After, Tower3]) :-  
    Tower2Before = [H|T],  
    Tower2After = T,  
    Tower1Before = L,  
    Tower1After = [H|L].  
  
m23([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-  
    Tower2Before = [H|T],  
    Tower2After = T,  
    Tower3Before = L,  
    Tower3After = [H|L].  
  
m31([Tower1Before, Tower2, Tower3Before], [Tower1After, Tower2, Tower3After]) :-  
    Tower3Before = [H|T],  
    Tower3After = T,  
    Tower1Before = L,  
    Tower1After = [H|L].  
  
m32([Tower1, Tower2Before, Tower3Before], [Tower1, Tower2After, Tower3After]) :-  
    Tower3Before = [H|T],  
    Tower3After = T,  
    Tower2Before = L,  
    Tower2After = [H|L].  
  
% -----  
% --- valid_state(S) :: S is a valid state  
  
valid_state([A|[B|[C]]]) :- towerState(A), towerState(B), towerState(C).  
  
towerState([]).  
towerState([s]).  
towerState([s,m]).  
towerState([s,m,l]).  
towerState([s,l]).  
towerState([s,l,h]).  
towerState([s,h]).  
towerState([s,m,h]).  
towerState([m]).  
towerState([m,l]).
```

```
towerState([m,l,h]).
towerState([m,h]).
towerState([m]).
towerState([m,l]).
towerState([m,l,h]).
towerState([m,h]).
towerState([l]).
towerState([l,h]).
towerState([h]).
towerState([s,m,l,h]).
towerState([t]).
towerState([t,s]).
towerState([t,s,m]).
towerState([t,s,m,l]).
towerState([t,s,l]).
towerState([t,s,l,h]).
towerState([t,s,h]).
towerState([t,s,m,h]).
towerState([t,m]).
towerState([t,m,l]).
towerState([t,m,l,h]).
towerState([t,m,h]).
towerState([t,m]).
towerState([t,m,l]).
towerState([t,m,l,h]).
towerState([t,m,h]).
towerState([t,l]).
towerState([t,l,h]).
towerState([t,h]).
towerState([t,s,m,l,h]).

% -----
% --- solve(Start,Solution) :: succeeds if Solution represents a path
% --- from the start state to the goal state.
solve :-
    extend_path([[s,m,l,h],[],[[]]],[],Solution),
    write_solution(Solution).

extend_path(PathSoFar,SolutionSoFar,Solution) :-
    PathSoFar = [[[]],[],[s,m,l,h]]|_,
    % showr('PathSoFar',PathSoFar),
```

```
% showr('SolutionSoFar',SolutionSoFar),
Solution = SolutionSoFar.

extend_path(PathSoFar,SolutionSoFar,Solution) :-
    PathSoFar = [CurrentState|_],
    % showr('PathSoFar',PathSoFar),
    make_move(CurrentState,NextState,Move),
    % show('Move',Move),
    % show('NextState',NextState),
    not(member(NextState,PathSoFar)),
    valid_state(NextState),
    Path = [NextState|PathSoFar],
    Soln = [Move|SolutionSoFar],
    extend_path(Path,Soln,Solution).

% -----
% --- write_sequence_reversed(S) :: Write the sequence, given by S,
% --- expanding the tokens into meaningful strings.
write_solution(S) :-
    nl, write('Solution ...'), nl, nl,
    reverse(S,R),
    write_sequence(R),nl.

write_sequence([]).
write_sequence([H|T]) :-
    elaborate(H,E),
    write(E),nl,
    write_sequence(T).

elaborate(m12,Output) :-
    Output = 'Transfer a disk from tower 1 to tower 2.'.

elaborate(m13,Output) :-
    Output = 'Transfer a disk from tower 1 to tower 3.'.

elaborate(m21,Output) :-
    Output = 'Transfer a disk from tower 2 to tower 1.'.

elaborate(m23,Output) :-
    Output = 'Transfer a disk from tower 2 to tower 3.'.

elaborate(m31,Output) :-
    Output = 'Transfer a disk from tower 3 to tower 1.'.
```

```
elaborate(m32,Output) :-  
    Output = 'Transfer a disk from tower 3 to tower 2.'
```

```
% -----  
% --- Unit test programs
```

```
test_m12 :-  
    write('Testing: move_m12\n'),  
    TowersBefore = [[t,s,m,l,h],[],[ ]],  
    trace(' ','TowersBefore',TowersBefore),  
    m12(TowersBefore,TowersAfter),  
    trace(' ','TowersAfter',TowersAfter).
```

```
test_m12x :-  
    write('Testing: move_m12\n'),  
    TowersBefore = [[s,m,l,h],[],[t]],  
    trace(' ','TowersBefore',TowersBefore),  
    m12(TowersBefore,TowersAfter),  
    trace(' ','TowersAfter',TowersAfter).
```

```
test_m13 :-  
    write('Testing: move_m13\n'),  
    TowersBefore = [[t,s,m,l,h],[],[ ]],  
    trace(' ','TowersBefore',TowersBefore),  
    m13(TowersBefore,TowersAfter),  
    trace(' ','TowersAfter',TowersAfter).
```

```
test_m13x :-  
    write('Testing: move_m13\n'),  
    TowersBefore = [[s,m,l,h],[],[t]],  
    trace(' ','TowersBefore',TowersBefore),  
    m13(TowersBefore,TowersAfter),  
    trace(' ','TowersAfter',TowersAfter).
```

```
test_m21 :-  
    write('Testing: move_m21\n'),  
    TowersBefore = [[],[t,s,m,l,h],[ ]],  
    trace(' ','TowersBefore',TowersBefore),  
    m21(TowersBefore,TowersAfter),
```

```
    trace(' ', 'TowersAfter', TowersAfter).

test__m23 :-
    write('Testing: move_m23\n'),
    TowersBefore = [[],[t,s,m,l,h],[ ]],
    trace(' ', 'TowersBefore', TowersBefore),
    m23(TowersBefore, TowersAfter),
    trace(' ', 'TowersAfter', TowersAfter).

test__m31 :-
    write('Testing: move_m31\n'),
    TowersBefore = [[],[ ],[t,s,m,l,h]],
    trace(' ', 'TowersBefore', TowersBefore),
    m31(TowersBefore, TowersAfter),
    trace(' ', 'TowersAfter', TowersAfter).

test__m32 :-
    write('Testing: move_m32\n'),
    TowersBefore = [[],[ ],[t,s,m,l,h]],
    trace(' ', 'TowersBefore', TowersBefore),
    m32(TowersBefore, TowersAfter),
    trace(' ', 'TowersAfter', TowersAfter).

test__valid_state :-
    write('Testing: valid_state\n'),
    test__vs([[l,t,s,m,h],[ ],[ ]]),
    test__vs([[t,s,m,l,h],[ ],[ ]]),
    test__vs([[ ],[h,t,s,m],[l]]),
    test__vs([[ ],[t,s,m,h],[l]]),
    test__vs([[ ],[h],[l,m,s,t]]),
    test__vs([[ ],[h],[t,s,m,l]]).

test__vs(S) :-
    valid_state(S),
    write(S), write(' is valid. '), nl.

test__vs(S) :-
    write(S), write(' is invalid. '), nl.

test__write_sequence :-
    write('First test of write_sequence ...'), nl,
    write_sequence([m31,m12,m13,m21]),
    write('Second test of write_sequence ...'), nl,
```

```
write_sequence([m13,m12,m32,m13,m21,m23,m13]).
```